



Sharp Test Series — Sample Paper

Chapter Test

QUESTION PAPER CODE
SMPLTHY-2026-000114

MODEL ANSWERS & MARKING SCHEME

Total Questions 14	Subjects Mathematics	Maximum Marks 30	Time 1 Hour 30 Minutes
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For examiners

The model answer and the marking points for every question are given below. Award the same marks for a different but correct method — the marking points say what to check, not the exact words.

SECTION A — Multiple Choice Questions

[5]

Attempt **all** questions. Each question carries 1 mark.

Q.1 If $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, then $A^{-1} = ?$

[1]

Answer: (C) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$|A| = 1$, so $A^{-1} = \text{adj } A = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$.

Q.2 A matrix that equals its own transpose ($A = A^T$) is called:

[1]

Answer: (A) A symmetric matrix

A matrix with $A=A^T$ is called symmetric — meaning corresponding elements on either side of the main diagonal are equal ($a_{ij}=a_{ji}$); in a skew-symmetric matrix, $A=-A^T$ instead ($a_{ij}=-a_{ji}$).

Q.3 Let $A = \begin{bmatrix} 5 & 5\lambda & \lambda \\ 0 & \lambda & 5\lambda \\ 0 & 0 & 5 \end{bmatrix}$ with $\lambda \neq 0$ and $|A^{-2}| = 1/25$. Then $\lambda = \dots\dots\dots$

[1]

Answer: (D) $\pm 1/5$

A is upper triangular, so $|A| = 5 \cdot \lambda \cdot 5 = 25\lambda$. Then $|A^{-2}| = 1/|A|^2 = 1/(625\lambda^2) = 1/25$ gives $\lambda^2 = 1/25$, so $\lambda = \pm 1/5$.

Q.4 Let M be a 3 by 3 matrix such that M times the column (1, 0, 0) is the column (1, 2, 3), M times the column (0, 1, 0) is the column (0, 1, 2) and M times the column (0, 0, 1) is the column (-1, 1, 1). If M times the column (x, y, z) is the column (1, 7, 11), then $x + y + z$ equals :

[1]

Answer: (B) 5

Multiplying M by a standard unit column simply picks out the matching column of M, so M has columns (1, 2, 3), (0, 1, 2) and (-1, 1, 1). Solving the resulting linear system gives $x = 2$, $y = 2$, $z = 1$, so $x + y + z = 5$.

Q.5 If $A = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$ with $i = \sqrt{-1}$, then $A^{(-50)} = \dots\dots\dots$

[1]

Answer: (D) -I

Squaring gives $A^2 = -I$, so $A^4 = I$ and $A^{(-50)} = A^{(-50 + 52)} = A^2 = -I$.

SECTION B — Short Answer Questions

[8]

Attempt **all** questions. Each question carries 2 marks.

Q.6 If A is a square matrix such that $A^2 = I$, show that A is invertible and $A^{-1} = A$. Verify that $A =$

[2]

$\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ is such a matrix.

$A^2 = I$ means $A \cdot A = I$. Here the same matrix A multiplies A on either side to give I , so by the definition of inverse A is invertible and $A^{-1} = A$.

For $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$:

$$A^2 = \begin{bmatrix} 1(1) + 0(2) & 1(0) + 0(-1) \\ 2(1) + (-1)(2) & 2(0) + (-1)(-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

So this A satisfies $A^2 = I$, and $A^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$.

Marking scheme: $A \cdot A = I \Rightarrow A^{-1} = A$ by the definition of inverse — 1; $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for the given matrix — 1

Q.7 Find the inverse of $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ by elementary row transformations.

[2]

$|A| = 1(1 \times 1 - 4 \times 0) - (-2)(0 \times 1 - 4 \times 0) + 0 = 1 \neq 0$, so A^{-1} exists.

$$AA^{-1} = I: \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \rightarrow R_1 + 2R_2: \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - 8R_3 \text{ and } R_2 \rightarrow R_2 - 4R_3: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 2 & -8 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Hence } A^{-1} = \begin{bmatrix} 1 & 2 & -8 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}.$$

Marking scheme: $R_1 \rightarrow R_1 + 2R_2$ (or any equivalent first step) applied to A and to I — 1; $R_1 \rightarrow R_1 - 8R_3$ and $R_2 \rightarrow R_2 - 4R_3$ (or any equivalent sequence) giving $A^{-1} = \begin{bmatrix} 1 & 2 & -8 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$ — 1 [EXAMINER: a first row $[1 \ 2 \ 0]$ instead of $[1 \ 2 \ -8]$ (the entry 8 created by the first step overlooked) earns the first mark only.]

Q.8 Without solving, state with reason whether the equations $2x - 4y = 3$ and $-3x + 6y = 1$ can be solved by the method of inversion.

[2]

$$A = \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix}; |A| = 2 \times 6 - (-4)(-3) = 12 - 12 = 0.$$

A is singular, so A^{-1} does not exist and $X = A^{-1}B$ cannot be formed. Hence the method of inversion cannot be used.

(In fact the second equation is equivalent to $2x - 4y = -\frac{2}{3}$, so the two lines are parallel and the system has no solution.)

Marking scheme: $|A| = 0$ — 1; A^{-1} does not exist, so the method of inversion cannot be used — 1

Q.9 If $A^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, find $(AB)^{-1}$. **[2]**

$$\begin{aligned} (AB)^{-1} &= B^{-1}A^{-1} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3+0 & 6+1 \\ 2+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}. \end{aligned}$$

Marking scheme: $(AB)^{-1} = B^{-1}A^{-1} - 1$; $B^{-1}A^{-1} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} - 1$ [EXAMINER: $A^{-1}B^{-1} = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$ is the wrong order and earns 0.]

SECTION C — Short Answer Questions **[9]**

Attempt **all** questions. Each question carries 3 marks.

Q.10 The sum of two numbers is 14, and three times the first number exceeds twice the second number by 12. Form the matrix equation and find the numbers by the method of reduction. **[3]**

Let the numbers be x and y . Then $x + y = 14$ and $3x - 2y = 12$.

$$\begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 3R_1: \begin{bmatrix} 1 & 1 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ -30 \end{bmatrix}$$

$$-5y = -30 \Rightarrow y = 6; x + 6 = 14 \Rightarrow x = 8.$$

The numbers are 8 and 6.

Marking scheme: Matrix equation $\begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \end{bmatrix} - 1$; $R_2 \rightarrow R_2 - 3R_1$ (or any equivalent transformation) applied to both sides, giving $\begin{bmatrix} 1 & 1 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ -30 \end{bmatrix} - 1$; $x = 8, y = 6 - 1$

Q.11 Show that the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 3 & 4 \end{bmatrix}$ has no inverse, first by evaluating $|A|$ and then by attempting **[3]**

to find A^{-1} by elementary row transformations.

(i) $|A| = 1(20 - 21) - 2(8 - 7) + 3(6 - 5) = -1 - 2 + 3 = 0$. A is singular, so A^{-1} does not exist.

(ii) Suppose A^{-1} exists. Then $AA^{-1} = I$: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 3 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$R_2 \rightarrow R_2 - 2R_1 \text{ and } R_3 \rightarrow R_3 - R_1: \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2: \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

The third row of the matrix on the left is zero, so the third row of the product on the left is $[0 \ 0 \ 0]$, while the third row on the right is $[1 \ -1 \ 1]$. This is a contradiction, so the supposition is false: A^{-1} does not exist, which agrees with (i).

Marking scheme: $|A| = 0$, so A is singular and A^{-1} does not exist $- 1$; $R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - R_1$ and $R_3 \rightarrow R_3 - R_2$ (or any equivalent sequence) applied to A and to I , producing a zero row on the left $- 1$; zero row $\Rightarrow [0 \ 0 \ 0] = [1 \ -1 \ 1]$, a contradiction, so A^{-1} does not exist $- 1$ [EXAMINER: a script that only evaluates $|A| = 0$ and concludes that A^{-1} does not exist earns 1 mark in all (the first line); the other two marks need the row-transformation working of part (ii).]

Q.12 Solve the following equations by the method of reduction: $y + 2z = 5$, $x + y + z = 4$, $3x - y + 2z =$ **[3]**
13.

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 13 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2: \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 13 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 3R_1: \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + 4R_2: \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 21 \end{bmatrix}$$

$$7z = 21 \Rightarrow z = 3; y + 2(3) = 5 \Rightarrow y = -1; x + (-1) + 3 = 4 \Rightarrow x = 2.$$

$$x = 2, y = -1, z = 3.$$

Marking scheme: $R_1 \leftrightarrow R_2$ (to get a non-zero leading element) and $R_3 \rightarrow R_3 - 3R_1$, applied to both sides — 1; $R_3 \rightarrow R_3 + 4R_2$ giving the upper triangular form with $7z = 21$ — 1; $z = 3$, $y = -1$, $x = 2$ — 1 [EXAMINER: any correct sequence of row transformations that reaches an upper triangular form earns the first two marks.]

SECTION D — Long Answer Questions

[8]

Attempt **all** questions. Each question carries 4 marks.

Q.13 For $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, verify that $A (\text{adj } A) = (\text{adj } A) A = |A| I$. [4]

$$|A| = 1(2 - 0) - 2(0 - 1) + 0 = 2 + 2 = 4.$$

Cofactors:

$$A_{11} = +(1 \times 2 - 1 \times 0) = 2 \quad A_{12} = -(0 \times 2 - 1 \times 1) = 1 \quad A_{13} = +(0 \times 0 - 1 \times 1) = -1$$

$$A_{21} = -(2 \times 2 - 0 \times 0) = -4 \quad A_{22} = +(1 \times 2 - 0 \times 1) = 2 \quad A_{23} = -(1 \times 0 - 2 \times 1) = 2$$

$$A_{31} = +(2 \times 1 - 0 \times 1) = 2 \quad A_{32} = -(1 \times 1 - 0 \times 0) = -1 \quad A_{33} = +(1 \times 1 - 2 \times 0) = 1$$

$$\text{adj } A = \begin{bmatrix} 2 & -4 & 2 \\ 1 & 2 & -1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$A (\text{adj } A) = \begin{bmatrix} 2+2+0 & -4+4+0 & 2-2+0 \\ 0+1-1 & 0+2+2 & 0-1+1 \\ 2+0-2 & -4+0+4 & 2+0+2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$(\text{adj } A) A = \begin{bmatrix} 2+0+2 & 4-4+0 & 0-4+4 \\ 1+0-1 & 2+2+0 & 0+2-2 \\ -1+0+1 & -2+2+0 & 0+2+2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$|A| I = 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Hence $A (\text{adj } A) = (\text{adj } A) A = |A| I$.

Marking scheme: $|A| = 4$ — 1; $\text{adj } A = \begin{bmatrix} 2 & -4 & 2 \\ 1 & 2 & -1 \\ -1 & 2 & 1 \end{bmatrix}$ — 1; $A (\text{adj } A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ — 1; $(\text{adj } A) A =$

$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = |A| I$ — 1 [EXAMINER: one wrong cofactor earns $\frac{1}{2}$ in the adj A line and two or more earn 0.]

Q.14 The sum of three numbers is 9. The first number minus the second number plus twice the third number is 7. Three times the first number plus the second number minus the third number is 5. Find the numbers by the method of inversion.

[4]

Let the numbers be x, y, z : $x + y + z = 9$, $x - y + 2z = 7$, $3x + y - z = 5$.

$$AX = B \text{ with } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 7 \\ 5 \end{bmatrix}.$$

$|A| = 1(1 - 2) - 1(-1 - 6) + 1(1 + 3) = -1 + 7 + 4 = 10 \neq 0$, so A^{-1} exists.

Cofactors:

$$A_{11} = +((-1)(-1) - 2 \times 1) = -1 \quad A_{12} = -(1 \times (-1) - 2 \times 3) = 7 \quad A_{13} = +(1 \times 1 - (-1) \times 3) = 4$$

$$A_{21} = -(1 \times (-1) - 1 \times 1) = 2 \quad A_{22} = +(1 \times (-1) - 1 \times 3) = -4 \quad A_{23} = -(1 \times 1 - 1 \times 3) = 2$$

$$A_{31} = +(1 \times 2 - 1 \times (-1)) = 3 \quad A_{32} = -(1 \times 2 - 1 \times 1) = -1 \quad A_{33} = +(1 \times (-1) - 1 \times 1) = -2$$

$$\text{adj } A = \begin{bmatrix} -1 & 2 & 3 \\ 7 & -4 & -1 \\ 4 & 2 & -2 \end{bmatrix}, A^{-1} = (1/10) \begin{bmatrix} -1 & 2 & 3 \\ 7 & -4 & -1 \\ 4 & 2 & -2 \end{bmatrix}$$

$(\text{adj } A)B$ has entries $-1(9) + 2(7) + 3(5) = 20$, $7(9) - 4(7) - 1(5) = 30$ and $4(9) + 2(7) - 2(5) = 40$.

$$X = A^{-1}B = (1/10) \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

The numbers are 2, 3 and 4.

Marking scheme: Equations in matrix form $AX = B$ — $\frac{1}{2}$; $|A| = 10$ — $\frac{1}{2}$; $\text{adj } A = \begin{bmatrix} -1 & 2 & 3 \\ 7 & -4 & -1 \\ 4 & 2 & -2 \end{bmatrix}$ —

1; $X = A^{-1}B = (1/10) \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$ — 1; the numbers 2, 3, 4 — 1 [EXAMINER: one wrong cofactor earns $\frac{1}{2}$

in the $\text{adj } A$ line and two or more earn 0. The method of inversion is asked for; numbers found by any other method earn only the first $\frac{1}{2}$.]