



Sharp Test Series — Sample Paper

Chapter Test

QUESTION PAPER CODE
SMPLTHY-2026-000112

MODEL ANSWERS & MARKING SCHEME

Total Questions 14	Subjects Physics	Maximum Marks 30	Time 1 Hour 30 Minutes
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For examiners

The model answer and the marking points for every question are given below. Award the same marks for a different but correct method — the marking points say what to check, not the exact words.

SECTION A — Multiple Choice Questions

[5]

Attempt **all** questions. Each question carries 1 mark.

Q.1 The theorem of perpendicular axes, $I_z = I_x + I_y$, is applicable only to

[1]

Answer: (C) plane laminar bodies

The perpendicular-axes theorem holds only for plane (laminar) bodies, with the x and y axes in the plane and the z axis perpendicular to it.

Q.2 A body tied to a string is whirled in a vertical circle of radius r. The minimum speed it must have at the topmost point is

[1]

Answer: (A) $\sqrt{g r}$

At the top, in the limiting case the weight alone provides the centripetal force: $mg = m v^2/r$, so $v(\text{min}) = \sqrt{g r}$.

Q.3 If M is the mass of a body and K its radius of gyration about a given axis, the moment of inertia of the body about that axis is

[1]

Answer: (C) $M K^2$

The radius of gyration is defined through $I = M K^2$, i.e. $K = \sqrt{I/M}$.

Q.4 The kinetic energy of a body of moment of inertia I rotating with angular velocity ω is

[1]

Answer: (D) $\frac{1}{2} I \omega^2$

Rotational kinetic energy = $\frac{1}{2} I \omega^2$, the analogue of $\frac{1}{2} m v^2$.

Q.5 A stone is whirled in a vertical circle of radius 1.6 m. The minimum speed it must have at the highest point is ($g = 10 \text{ m/s}^2$)

[1]

Answer: (A) 4 m/s

$v(\text{top, min}) = \sqrt{g r} = \sqrt{10 \times 1.6} = \sqrt{16} = 4 \text{ m/s}$.

SECTION B — Short Answer Questions

[8]

Attempt **all** questions. Each question carries 2 marks.

- Q.6** Explain, using the principle of conservation of angular momentum, why a ballet dancer spins faster when she folds her arms close to her body. [2]

While spinning, the dancer touches the floor at essentially one point and the external torque about her vertical axis is very nearly zero. Hence her angular momentum $L = I\omega$ is conserved, i.e. $I_1\omega_1 = I_2\omega_2$.

When she folds her arms (and stretched leg) close to the axis of rotation, the mass of her body comes nearer to the axis. Since $I = \sum m r^2$, her moment of inertia decreases.

As the product $I\omega$ must stay constant, a decrease in I is compensated by an increase in ω , so she spins faster. Stretching the arms out again raises I and slows her down.

Marking scheme: Zero external torque $\Rightarrow I\omega$ constant — 1; folding arms reduces r hence I , so ω increases — 1

- Q.7** Distinguish between centripetal force and centrifugal force on any two points. [2]

Centripetal force: (i) it is a real force, supplied by a physical agent — tension, friction, gravitation or the normal reaction; (ii) it is directed towards the centre of the circular path; (iii) it exists in an inertial frame of reference.

Centrifugal force: (i) it is a pseudo (fictitious) force with no physical agent and no reaction; (ii) it is directed away from the centre; (iii) it has to be introduced only in the rotating, non-inertial frame of the revolving body.

Marking scheme: Any two correct points of difference — 1 each

- Q.8** The blades of a ceiling fan make 300 revolutions per minute. Calculate the angular speed of the fan and its period of revolution. [2]

Data: 300 revolutions per minute

Frequency $n = 300/60 = 5 \text{ rev s}^{-1}$

Angular speed $\omega = 2\pi n = 2 \times 3.142 \times 5 = 31.42 \text{ rad s}^{-1}$

Period $T = 1/n = 1/5 = 0.2 \text{ s}$

Marking scheme: $n = 5 \text{ rev s}^{-1}$ and $\omega = 2\pi n = 31.42 \text{ rad s}^{-1}$ — 1; $T = 1/n = 0.2 \text{ s}$ — 1

- Q.9** Write the moments of inertia of a solid sphere and of a thin hollow sphere, each of mass M and radius R , about a diameter. State with reason which of the two is greater. [2]

Solid sphere about a diameter: $I = (2/5) M R^2 = 0.4 M R^2$

Thin hollow sphere (spherical shell) about a diameter: $I = (2/3) M R^2 \approx 0.667 M R^2$

The hollow sphere has the greater moment of inertia. In a hollow sphere the whole of the mass lies on the surface, at the maximum possible distance R from the centre, whereas in a solid sphere much of the mass lies closer to the axis. Since $I = \sum m r^2$, mass placed farther from the axis contributes more, so I is larger for the hollow sphere.

Marking scheme: Both expressions $(2/5)MR^2$ and $(2/3)MR^2$ — 1; hollow sphere greater, because its mass lies farthest from the axis — 1

SECTION C — Short Answer Questions

[9]

Attempt **all** questions. Each question carries 3 marks.

- Q.10** Derive the expression for the kinetic energy of a body rotating with a uniform angular velocity about a fixed axis. [3]

Consider a rigid body rotating about a fixed axis with uniform angular velocity ω . Imagine the body to be made up of N particles of masses $m_1, m_2, \dots, m_N$ at perpendicular distances $r_1, r_2, \dots, r_N$ from the axis.

Every particle describes a circle about the axis in the same time, so all of them have the same angular velocity ω , but their linear speeds differ: $v_i = r_i \omega$.

Kinetic energy of the i -th particle $= \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$

Total kinetic energy of the body $= \sum \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2} \omega^2 \sum m_i r_i^2$

But $\sum m_i r_i^2 = I$, the moment of inertia of the body about that axis. Hence

$$E_{\text{rot}} = \frac{1}{2} I \omega^2$$

This is the rotational analogue of $\frac{1}{2} m v^2$: I replaces m and ω replaces v .

Marking scheme: Body as particles, all with the same ω and $v_i = r_i \omega$ — 1; summation $\frac{1}{2} \omega^2 \sum m_i r_i^2$ — 1; identification of $\sum m_i r_i^2$ as I giving $E = \frac{1}{2} I \omega^2$ — 1

- Q.11** A uniform rod of mass 3 kg and length 1.2 m rotates about an axis perpendicular to its length. Calculate its moment of inertia when the axis passes through (a) its centre and (b) one of its ends. Also find its radius of gyration in case (b). [3]

Data: $M = 3$ kg, $L = 1.2$ m, so $L^2 = 1.44$ m²

(a) Axis through the centre, perpendicular to the length:

$$I_c = M L^2 / 12 = (3 \times 1.44) / 12 = 4.32 / 12 = 0.36 \text{ kg m}^2$$

(b) Axis through one end, perpendicular to the length. By the theorem of parallel axes with $h = L/2 = 0.6$ m:

$$I_{\text{end}} = I_c + M h^2 = 0.36 + 3 \times (0.6)^2 = 0.36 + 1.08 = 1.44 \text{ kg m}^2$$

(This agrees with the standard result $I_{\text{end}} = M L^2/3 = 4.32/3 = 1.44 \text{ kg m}^2$.)

Radius of gyration in case (b):

$$k = \sqrt{I_{\text{end}} / M} = \sqrt{1.44 / 3} = \sqrt{0.48} = 0.693 \text{ m}$$

Marking scheme: $I_c = ML^2/12 = 0.36 \text{ kg m}^2$ — 1; parallel-axis step with $h = 0.6$ m giving $I_{\text{end}} = 1.44 \text{ kg m}^2$ — 1; $k = \sqrt{I/M} = 0.693 \text{ m}$ with unit — 1

- Q.12** A small body tied to the end of a string of length 0.9 m is revolved in a vertical circle. Calculate (a) the minimum speed it must have at the highest point to complete the circle, and (b) its speed at the lowest point in that case. ($g = 9.8 \text{ m s}^{-2}$) [3]

Data: $r = 0.9$ m, $g = 9.8 \text{ m s}^{-2}$

(a) At the highest point the string is just slack ($T = 0$) and gravity alone provides the centripetal force:

$$m g = m v_{\text{top}}^2 / r \Rightarrow v_{\text{top}} = \sqrt{g r} = \sqrt{9.8 \times 0.9} = \sqrt{8.82}$$

$$v_{\text{top}} = 2.97 \text{ m s}^{-1}$$

(b) Taking the lowest point as the reference level and using conservation of energy between the lowest and the highest point (height difference $2r$):

$$\frac{1}{2} m v_{\text{bot}}^2 = \frac{1}{2} m v_{\text{top}}^2 + m g (2r)$$

$$v_{\text{bot}}^2 = v_{\text{top}}^2 + 4 g r = g r + 4 g r = 5 g r = 5 \times 9.8 \times 0.9 = 44.1$$

$$v_{\text{bot}} = \sqrt{44.1} = 6.64 \text{ m s}^{-1}$$

Marking scheme: Condition $T = 0$ and $v_{\text{top}} = \sqrt{gr}$ — 1; $v_{\text{top}} = 2.97 \text{ m s}^{-1}$ — 1; energy equation giving $v_{\text{bot}} = \sqrt{5gr} = 6.64 \text{ m s}^{-1}$ with unit — 1

SECTION D — Long Answer Questions**[8]**Attempt **all** questions. Each question carries 4 marks.

- Q.13** A body of mass 0.2 kg tied to a string of length 1.0 m is revolved in a vertical circle. At the highest point its speed is 1.5 times the minimum speed needed there. Calculate (a) its speed at the highest point, (b) the tension in the string at the highest point, (c) its speed at the lowest point and (d) the tension at the lowest point. ($g = 9.8 \text{ m s}^{-2}$) **[4]**

Data: $m = 0.2 \text{ kg}$, $r = 1.0 \text{ m}$, $g = 9.8 \text{ m s}^{-2}$ (a) Minimum speed at the highest point: $v_{\min} = \sqrt{g r} = \sqrt{9.8 \times 1.0} = 3.13 \text{ m s}^{-1}$ Given speed at the highest point: $v_1 = 1.5 \times 3.13 = 4.70 \text{ m s}^{-1}$ (so $v_1^2 = 2.25 \times g r = 2.25 \times 9.8 = 22.05 \text{ m}^2 \text{ s}^{-2}$)(b) At the highest point: $T_1 + m g = m v_1^2 / r$ $T_1 = m v_1^2 / r - m g = (0.2 \times 22.05) / 1.0 - (0.2 \times 9.8) = 4.41 - 1.96$ $T_1 = 2.45 \text{ N}$ (c) Conservation of energy between the highest and the lowest point (height difference $2r = 2.0 \text{ m}$): $v_2^2 = v_1^2 + 4 g r = 22.05 + (4 \times 9.8 \times 1.0) = 22.05 + 39.2 = 61.25 \text{ m}^2 \text{ s}^{-2}$ $v_2 = \sqrt{61.25} = 7.83 \text{ m s}^{-1}$ (d) At the lowest point: $T_2 - m g = m v_2^2 / r$ $T_2 = m v_2^2 / r + m g = (0.2 \times 61.25) / 1.0 + 1.96 = 12.25 + 1.96$ $T_2 = 14.21 \text{ N}$ Check: $T_2 - T_1 = 14.21 - 2.45 = 11.76 \text{ N} = 6 m g = 6 \times 0.2 \times 9.8 = 11.76 \text{ N} \checkmark$

Marking scheme: (a) $v_{\min} = \sqrt{g r} = 3.13 \text{ m s}^{-1}$ and $v_1 = 4.70 \text{ m s}^{-1} - 1$; (b) $T_1 = m v_1^2 / r - m g = 2.45 \text{ N} - 1$; (c) $v_2^2 = v_1^2 + 4 g r$ giving $v_2 = 7.83 \text{ m s}^{-1} - 1$; (d) $T_2 = m v_2^2 / r + m g = 14.21 \text{ N} - 1$

Q.14 State the theorem of parallel axes and the theorem of perpendicular axes for moment of inertia, [4]
and prove any one of them.

(Both theorems are to be stated; only ONE proof is expected from the student. Both proofs are given below for the examiner's reference.)

THEOREM OF PARALLEL AXES

Statement: The moment of inertia of a body about any axis is the sum of its moment of inertia about a parallel axis through its centre of mass and the product of its mass and the square of the perpendicular distance between the two axes: $I_O = I_c + M h^2$.

Proof: Let the given axis pass through the point O and let the parallel axis pass through the centre of mass C, the perpendicular distance OC between them being h. Consider a particle of the body of mass dm at a point D. Drop DN perpendicular to the line OC (produced if necessary).

Moment of inertia of the body about the axis through C: $I_c = \int (DC)^2 dm$

Moment of inertia about the given axis through O: $I_O = \int (DO)^2 dm$

From the right-angled triangle DNO:

$$(DO)^2 = (DN)^2 + (NO)^2 = (DN)^2 + (NC + h)^2$$

$$= (DN)^2 + (NC)^2 + h^2 + 2h(NC)$$

But from the right-angled triangle DNC, $(DN)^2 + (NC)^2 = (DC)^2$. Hence

$$(DO)^2 = (DC)^2 + h^2 + 2h(NC)$$

Multiplying by dm and integrating over the whole body:

$$I_O = \int (DC)^2 dm + h^2 \int dm + 2h \int (NC) dm$$

The first term is I_c and $\int dm = M$, so the second term is $M h^2$. In the third term, NC is the distance of the particle from the centre of mass measured along the line OC, so $\int (NC) dm$ is the sum of the moments of all the masses about the centre of mass — and this is zero, by the very definition of the centre of mass.

$$\text{Hence } I_O = I_c + M h^2.$$

THEOREM OF PERPENDICULAR AXES

Statement: The moment of inertia of a plane lamina about an axis perpendicular to its plane is the sum of its moments of inertia about two mutually perpendicular axes lying in its plane, all three axes being concurrent: $I_z = I_x + I_y$.

Proof: Let a plane lamina lie in the XY-plane, with the x- and y-axes in its plane and the z-axis perpendicular to it, all meeting at O. Consider a particle of mass dm at the point P(x, y) of the lamina, at a distance r from O, so that $r^2 = x^2 + y^2$.

Its perpendicular distance from the x-axis is y and from the y-axis is x, while its perpendicular distance from the z-axis is r. Therefore

$$I_x = \int y^2 dm, I_y = \int x^2 dm, I_z = \int r^2 dm$$

$$I_z = \int (x^2 + y^2) dm = \int x^2 dm + \int y^2 dm = I_y + I_x$$

$$\text{Hence } I_z = I_x + I_y.$$

This theorem holds only for a plane lamina, because it needs every particle to lie in the XY-plane so that $r^2 = x^2 + y^2$.

Marking scheme: Statement of the theorem of parallel axes with $I_O = I_c + Mh^2$ — 1; statement of the theorem of perpendicular axes with $I_z = I_x + I_y$ (plane lamina, concurrent axes) — 1; proof of any ONE of the two theorems, complete with the construction and the key step (the vanishing of $\int (NC)dm$ at the centre of mass, or $r^2 = x^2 + y^2$ for the lamina) — 2